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Multivariable Adaptive Control of the Rewinding Process of a Roll-to-roll System Governed by Hyperbolic Partial Differential Equations

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Abstract: In this paper, an active control scheme for the rewinding process of a roll-to-roll (R2R) system is investigated. The control objectives are to suppress the transverse vibration of the moving web, to track the desired velocity profile, and to keep the desired radius value of a rewind roller. The bearing coefficient in the rewind shaft is unknown and the rotating elements in the drive motor are various. The moving web is modeled as an axially moving beam system governed by hyperbolic partial differential equations (PDEs). The control scheme utilizes two control inputs: a control force exerted from a hydraulic actuator equipped with a damper, and a control torque applied to the rewind roller. Two adaptation laws are derived to estimate the unknown bearing coefficient and the bound of variations of the rotating elements. The Lyapunov method is employed to prove the robust stability of the rewind section, specifically the uniform and ultimate boundedness of all of the signals. The effectiveness of the proposed control schemes was verified by numerical simulations.

Keywords: Adaptive boundary control, axially moving beam, hyperbolic partial differential equation, Lyapunov method, roll-to-roll system, velocity tracking.

1. INTRODUCTION

There are many industries that use web-material transport systems, for example, paper, textile, metal, polymer, composite, etc. In these systems, the roll-to-roll (R2R) processing technique yields a better performance by allowing mass production with high-speed automation [1]. A number of researchers have investigated the dynamics and control problems of R2R systems, focusing on tension and velocity controls for web handling [2–15]. However, most these studies were based on a dynamic model in the form of ordinary differential equations (ODEs) and perfect knowledge on the system parameters. Typically, Pagillar *et al*. [9] used a decentralized model reference adaptive control scheme to deal with the radius change of the unwind/rewind rolls and disturbances for a large-scale R2R system. Moreover, it is obvious that the mechanical vibration problem of moving webs has not received in-depth attention, though such vibrations (particularly in the transverse direction) of the moving web are the main quality- and productivity-limiting factors, especially for a high-speed R2R system [16, 17]. Specifically, in the rewind process, the quality of the wound roll is affected by the transverse vibrations and the transport velocity of the moving web. Thus, motivated from the industry problems, this paper addresses the issues of the transverse vibration control and the transport velocity control in the rewinding process of a high-speed R2R system. By definition, R2R systems are those that provide continuous axial transports of flexible materials. They belong to the class of axially moving systems that are modeled by partial differential equations (PDEs). It should be noted here that PDE models can consider transverse vibrations only.

Vibration control schemes for axially moving systems [16–30], flexible string/beam systems [31–39] and other systems [40–42] have been extensively investigated in the literature. Using the Lyapunov energy-based method for distributedd parameter systems, boundary control laws have been derived. A boundary control law uses measured signals of transverse displacement and the time-rate of the slope of the moving material at the left or right boundary, whose data are obtainable by the addition of laser sensors at boundary points. Therefore, insofar as actuators and sensors can be easily assembled at a boundary, the boundary control method can provide a practical control solution for axially moving systems. Boundary control can be im-

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plemented in two ways: active control [16–19, 21–26, 28–30, 33–38] or, with a proper damping mechanism, passive (i.e., semi-active) control [20, 27].

Adaptive control for axially moving systems [18, 20–22, 24, 26, 28] and flexible beam systems [33, 36, 37] has been an interesting issue for many researchers. In these works, indirect adaptive control schemes have been used. In [18], estimation laws of actuator mass and tension of flexible material were developed. Nguyen and Hong [28] proposed a boundary control using adaptation laws for mass per unit length of a moving string, mass of actuator, viscous damping coefficient, and disturbance. He and Zhang [37] developed a vibration control algorithm for a flexible robot under presence of input deadzone, where a neural network has been employed to compensate the effect of the input deadzone.

In this paper, we introduce a dynamic model (using hybrid ODE-PDEs) for the rewinding process of a R2R system including moving-web and rewind-roll dynamics, where the moving web is modeled as an axially moving beam. In comprarison with the model derived in [9], the proposed dynamic model of the rewinding process provides more advantages in analyzing the vibrations of the moving material together with the transport velocity and in taking into account the variation of the radius of the rewind roller.

In control design, based on the dynamic models, the Lyapunov method is employed to derive a boundary control law for reduction of the transverse vibrations of the moving web under the following assumption: The transport velocity and the tension are slowly time-varying parameters in the model. For velocity control of the rewind roller under the effects of an unknown bearing friction coefficient in the rewind shaft and variations of the rotating elements on the drive motor side, an adaptive control scheme is proposed. It should be noted that the radius of the rewinding process also needs to be driven to the desired value. Therefore, an adaptive control scheme for the rewind roller is designed to regulate the rewind roller in order to achieve the desired velocity and the desired radius simultaneously.

The contributions of this paper are as follows: First, vibration suppression of the rewind section of the R2R system is considered under the dynamic effects of transport velocity and time-varying tension. By vibration suppression, the performance of the rewind process is improved, especially in the case of high-speed R2R systems wherein the transport velocity and time-varying tension excite a large amplitude of the web material [30]. Second, having considered the variation of the radius of the rewind roller in the rewind roller dynamics, a control torque law is developed for control of the velocity and radius of the rewind roller under the effects of an unknown bearing friction coefficient in the rewind shaft and the variations of the rotating elements on the drive motor side.

2. PROBLEM FORMULATION

Fig. 1 shows a schematic of the rewind section with a hydraulic actuator for vibration suppression of the moving web, where t is the time, x is the spatial coordinate along the longitude of motion, $v(t)$ is the time-varying transport velocity of the web, $w(x,t)$ is the transverse displacement of the web, and l is the distance between the fixed rolls and the touch rolls. The parameters describing the material properties of the web are the mass per unit length ρ, the cross-section area A, the Young modulus E, the moment of inertia of the web cross section I, and the viscous damping coefficient c_v. The parameters of the hydraulic actuator are the mass of the touch rolls m_a and the damping coefficient c_a. If we let $T(x,t)$ denote the spatially varying tension, control force $f_a(t)$ is applied to the touch rolls to suppress the transverse vibrations of the moving web. For notational convenience, instead of $w_x(x,t)$ and $w_t(x,t)$, w_x and w_t will be used, and similar abbreviations are employed subsequently.

As shown in Fig. 1(a), the hydraulic actuator is located near the rewind roller; thus, the distance between the touch rolls and the rewind roll is very small in comparison to the distance between the touch rolls and the fixed rolls. Therefore, it is assumed that the transverse vibrations occur only in the web span between the fixed rolls and the touch rolls. Based on this assumption, along the span, the moving web is modeled as an axially moving beam, where the touch rolls are considered as the right boundary. The governing equation and boundary conditions of the system in Fig. 1 are given as follows:

$$\rho A(w_{tt} + \dot{v}w_x + 2vw_{xt} + v^2 w_{xx}) - (Tw_x)_x + c_v(w_t + vw_x) + EIw_{xxxx} = 0, \quad (1)$$

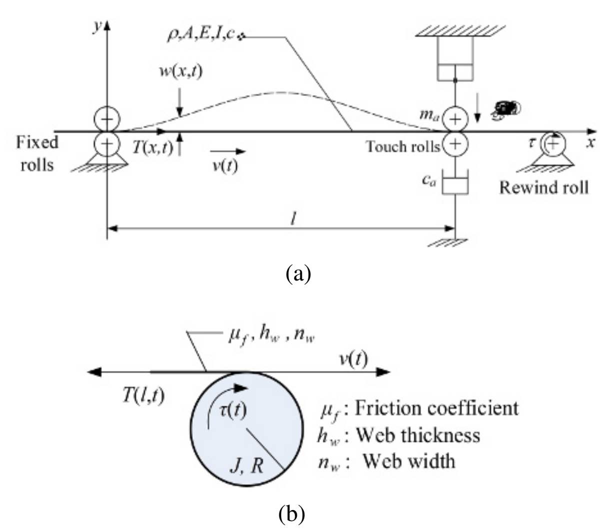

Fig. 1. The schematic of a R2R system: (a) Rewind section and (b) rewind roller.

$$w(x,0) = w_0(x), \;\; w_t(x,0) = w_{t_0}(x), \tag{2}$$
$$w(0,t) = 0, \;\; w_x(l,t) = 0, \;\; w_{xx}(l,t) = 0, \tag{3}$$
$$m_a w_{tt}(l,t) + c_a w_t(l,t) + T(l,t) w_x(l,t) - EI w_{xxxx}(l,t) - f_a(t) = 0. \tag{4}$$

Note that (1) governs the transverse displacement of the moving web. The initial conditions are given in (2), and the boundary conditions are given by (3) and (4). Equation (4) also describes the dynamics of the hydraulic actuator. In this web span, the tension $T(x,t)$ is assumed to be spatially varying and is given as [23]

$$T(x,t) = T_0 + \rho A(l-x)\dot{v}, \tag{5}$$

where T_0 denotes the initial tension applied to the web. In the control design presented in the next section, the tension variation will be incorporated. From (5), it is shown that the spatially varying tension is continuous and uniformly bounded for all $x \in [0,l]$ and $t \in [0,\infty)$.

Fig. 1(b) shows the cross-sectional view of the rewind roller as driven by an electric motor. Let h_w be the web thickness and n_w be the web width; then, J and R denote the rewind roller's inertia and radius, respectively. The bearing friction coefficient in the rewind roll shaft μ_f is assumed to be unknown. The variations of the drive-motor-side rotating elements, which include the motor armature, the driving pulley (or gear) and the driving shaft, among others, are represented as disturbance $\delta(t)$, which is bounded by an unknown positive constant δ_d. The motor torque $\tau(t)$ is applied to the rewind roll to maintain its velocity levels and to wind the flexible materials. It is assumed that the mass of the touch rollers is small. Therefore, the tension of the web coming into the rewind roller is $T(l,t) = T_0$.

The time-varying radius of the rewind roller is given as

$$R(t) = R_0 + \frac{h_w s(t)}{2\pi R(t)}, \tag{6}$$

where R_0 is the radius of the driven shaft, and $s(t)$ is the length of the wounded materials. From (6), the length of the flexible material can be obtained as

$$s(t) = \frac{2\pi}{h_w}(R^2(t) - R(t)R_0). \tag{7}$$

The dynamics of the radius of the rewind roller is derived as

$$\dot{R}(t) = \frac{h_w v(t)}{(2\pi+1)R - R_0}. \tag{8}$$

The inertia of the rewind roller is given as

$$J(t) = \frac{1}{2}(m_0 + \rho h_w n_w s(t))R^2(t) = \frac{1}{2}m_0 R^2 + \pi\rho n_w R^4 + \pi\rho n_w R_0 R^3, \tag{9}$$

where m_0 is the mass of the core of the rewind roller. The velocity dynamics of the rewind roller are given as [7]

$$\frac{J}{R}\dot{v} = -\frac{\mu_f}{R}v + \frac{J\dot{R}}{R^2}v - 2\pi\rho n_w R^2 \dot{R} v - T(l,t)R + \delta(t) + \tau(t). \tag{10}$$

Thus, using (7) and (8), (10) can be rewritten as

$$\begin{aligned}\dot{v}(t) = &-\frac{\mu_f m_0}{2}R^2(t)v(t) + \frac{1}{R(t)}\frac{h_w}{(2\pi+1)R - R_0}v^2(t) \\ &- \frac{2\pi\rho n_w h_w R^2(t)}{(2\pi+1)R - R_0}v^2(t) \\ &- \frac{T_0}{m_0 + 2\pi\rho n_w R^2 + 2\pi\rho n_w R_0 R} \\ &+ \frac{2}{m_0 + 2\pi\rho n_w R^3 + 2\pi\rho n_w R_0 R^2}\tau(t) \\ &+ \frac{1}{m_0 + 2\pi\rho n_w R^3 + 2\pi\rho n_w R_0 R^2}\delta(t).\end{aligned} \tag{11}$$

It is noted that the hybrid ODE-PDE model including (1)-(4), (8) and (11) represents the rewinding process [29,30]. This dynamic model considers the transverse vibrations, the time-varying radius, and the time-varying transport velocity.

3. CONTROL DESIGNS

In this section, the control objectives are to suppress the transverse vibrations of the moving web, to maintain the required level of the transport velocity, and to drive the radius of the rewind roller to reach the desired value. To achieve these objectives, the control scheme includes two control inputs, which are exerted from the hydraulic actuator for transverse vibration suppression and from the drive motor of the rewind roller for transport velocity regulation and radius control. In the beam model (1), the transport velocity is assumed to be time-varying and is incorporated into the vibration control design. In practice, it should be noted that not only the value of the transport-velocity tracking error but also the time to eliminate the error affects the transverse vibration.

The tracking errors of the transport-velocity and radius controls are defined, respectively, as

$$e_v = v(t) - v_d(t), \tag{12}$$
$$e_R = R(t) - R_d, \tag{13}$$

where v_d is the desired pattern of the velocity, and R_d is the desired radius. Subtituting (12)-(13) into (8)-(11), we obtain the following error dynamics.

$$\dot{e}_R = \frac{h_w(e_v + v_d)}{(2\pi+1)(e_R + R_d) - R_0}, \tag{14}$$

$$\dot{e}_v = -\frac{\mu_f m_0}{2}(e_R + R_d)^2 v - \frac{2\pi\rho n_w h_w (e_R + R_d)^2}{(2\pi+1)(e_R + R_d) - R_0}v^2$$

$$+\frac{1}{m_0+2\pi\rho n_w(e_R+R_d)^2(e_R+R_d+R_0)}\delta(t) - \frac{T_0}{m_0+2\pi\rho n_w(e_R+R_d)(e_R+R_d+R_0)} + \frac{2}{m_0+2\pi\rho n_w(e_R+R_d)(e_R+R_d+R_0)}\tau(t). \tag{15}$$

From (1), it is shown that the transport velocity and its rate of change can affect the transverse vibrations. This effect is especially significant in the case where the transport velocity is controlled to track high-slope patterns due to the high rate of change of the velocity. As shown in (11) and (15), the variation of the rotating elements in the drive motor and the variation of the radius of the rewind roller can disturb the velocity dynamics as well as the velocity tracking error.

In control design then, the unknown bearing friction coefficient μ_f and the bound of disturbance δ_d are estimated. Let $\hat{\mu}_f(t)$ and $\hat{\delta}_d(t)$ be the estimated values of μ_f and δ_d, respectively. The estimation errors $\tilde{\mu}(t)$ and $\tilde{\delta}_d(t)$, respectively, are defined as follows:

$$\tilde{\mu}(t)=\hat{\mu}_f(t)-\mu_f, \tag{16}$$

$$\tilde{\delta}_d(t)=\hat{\delta}_d(t)-\delta_d(t). \tag{17}$$

Based on the total mechanical energy of the rewinding process, a Lyapunov function candidate is introduced as follows.

$$V(t)=\alpha E_{\text{beam}}(t)+E_{\text{act}}(t)+V_{\text{cross}}(t)+V_{rw}(t)+V_{\text{est}}(t), \tag{18}$$

where the moving web energy, the actuator energy, the cross term, the tracking error, and the estimate term are defined individually as follows.

$$E_{\text{beam}}(t)=\frac{1}{2}\int_0^l \rho(w_t+vw_x)^2dx + \frac{1}{2}\int_0^l (Tw_x^2+EIw_{xx}^2)dx, \tag{19}$$

$$E_{\text{act}}(t)=\frac{1}{2}m_a\left\{\alpha w_t(l,t)+(v(t)+2\beta l)w_x(l,t)\right\}^2, \tag{20}$$

$$V_{\text{cross}}(t)=2\beta\int_0^l xw_x(w_t+v(t)w_x)dx, \tag{21}$$

$$V_{rw}(t)=\frac{1}{2}(e_R^2+e_v^2), \tag{22}$$

$$V_{\text{est}}(t)=\frac{1}{2}(\tilde{\mu}_d^2+\tilde{\delta}_d^2), \tag{23}$$

where α and β are positive constants. It should be noted that $V_{\text{cross}}(t)$ is not positive definite. Consider the following function

$$V_m(t)=V(t)-E_{\text{act}}(t)-V_{rw}(t)-V_{\text{est}}(t). \tag{24}$$

It should be noted that if $\alpha \geq 2\beta l$, the following inequalities hold.

$$\frac{\alpha}{2}\int_0^l \rho(w_t+vw_x)^2dx \geq \beta\int_0^l \rho l(w_t+vw_x)^2dx \geq \beta\int_0^l \rho x(w_t+vw_x)^2dx, \tag{25}$$

$$\frac{\alpha}{2}\int_0^l Tw_x^2dx \geq \frac{\alpha T_{\min}}{2}\int_0^l w_x^2dx \geq \beta\int_0^l \rho lw_x^2dx \geq \beta\int_0^l \rho xw_x^2dx. \tag{26}$$

Then, using the Cauchy-Schwarz inequality, the following inequalities are obtained.

$$(\alpha-\psi)E_{\text{beam}} \leq V_m(t) \leq (\alpha+\psi)E_{\text{beam}}, \tag{27}$$

where

$$\psi=\max\left\{\frac{2\beta l}{T_{\min}},2\beta l\right\}. \tag{28}$$

It can be seen that $V_m(t)$ and consequently $V(t)$ are positive-definite functions if

$$\alpha \geq \psi. \tag{29}$$

The control input law exerted from the hydraulic actuator is introduced as follows:

$$f_a(t)=-m_a\left\{\dot{v}(t)w_x(l,t)+(\alpha v(t)+2\beta l)w_{xt}(l,t)\right\} + c_aw_t(l,t)-\frac{2\beta Alv(t)}{\alpha v(t)+2\beta l}w_t(l,t)\rho. \tag{30}$$

The control torque applied to the driven shaft of the rewind roller is given as follows.

$$\begin{aligned}\tau(t)=&\left[m_0+2\pi\rho n_w(e_R+R_d)^2(e_R+R_d+R_0)\right]\\&\times\left[\frac{\hat{\mu}_fm_0}{2}(e_R+R_d)^2v_d\right.\\&+\frac{1}{(e_R+R_d)}\frac{h_w}{(2\pi+1)(e_R+R_d)-R_0}(e_v+v_d)^2\\&\times\frac{T_0}{m_0+2\pi\rho n_w(e_R+R_d)(e_R+R_d+R_0)}\\&\left.+\frac{2\pi\rho n_wh_w(e_R+R_d)^2(t)}{(2\pi+1)(e_R+R_d)-R_0}(e_v+v_d)^2-k_ve_v^2\right]\\&-H(e_v)\hat{\delta}_d(t),\end{aligned} \tag{31}$$

where k_v is the control gain, and the differentiable signum function $H(e_v)$ is given as follows [31]:

$$H(e_v)=2\frac{\int_{-0.01}^{e_v}u(x+0.01)u(0.01-x)dx}{\int_{-0.01}^{0.01}u(x+0.01)u(0.01-x)dx}-1, \tag{32}$$

where

$$u(y)=\begin{cases}0, & \text{if } y\leq 0,\\ y, & \text{if } y>0.\end{cases} \tag{33}$$

This control law includes the estimates $\hat{\mu}_f(t)$ and $\hat{\delta}_d(t)$, which are given by the following adaptation laws.

$$\dot{\hat{\mu}}_f(t) = -k\mu_f(t) - \frac{m_0}{2}(e_R + R_d)^2 v_d e_v, \tag{34}$$

$$\dot{\hat{\delta}}_d = -k_\delta \delta_d + \frac{1}{M_0 + 2\pi\rho n_w (e_R + R_d)^2 (e_R + R_d + R_0)}, \tag{35}$$

where k_μ and k_δ are the adaptation gains.

The control law (33) requires the measurement of the transverse displacement $w(l,t)$ and the slope of the web $w_x(l,t)$. The transverse displacement $w(l,t)$ can be measured using a displacement sensor attached to the hydraulic actuator; the slope of the web $w_x(l,t)$, meanwhile, can be obtained using a laser sensor at the right boundary [21–25]. The backward differencing of such signals can provide $w_t(l,t)$ and $w_{xt}(l,t)$.

The control law (31) can be implemented with the measurements of the radius of the rewind roller and the transport velocity, which can be measured by using a laser sensor and an encoder attached to the motor driving the rewind roller.

Lemma 1 [29]**:** Given $u(x,t) : [0,l] \times R^+ \to R$, if $u(0,t) = 0$,

$$u^2(x,t) \le l \int_0^l u_x^2(x,t)dx, \tag{36}$$

$$\int_0^l u^2(x,t)dx \le l^2 \int_0^l u_x^2(x,t)dx. \tag{37}$$

Property 1: If the energy of the axially moving beam E_{beam} is bounded, then w_{xt} is bounded [39].

Theorem 1: Consider the rewinding system represented by (1)-(4), (8) and (11). Then, the control laws (30) and (31) using adaptation laws (34) and (35) guarantee the robust stability of the closed-loop system in the sense that all signals i.e., the transverse vibration, tracking errors, and estimation errors) are uniformly and ultimately bounded.

Proof: The time derivative of the Lyapunov function candidate (18) results in

$$\begin{aligned}\dot{V}(t) \le& -\beta\left\{c_v l + \rho A v - \frac{\alpha}{\beta}v - \rho A\right\}\int_0^l (w_t + v w_x)^2 dx \\ &- \frac{\alpha\hat{T}}{2\max_{x\in[0,l]}\{T\}}\int_0^l T w_x^2 dx - \frac{\alpha EI}{2}\int_0^l w_{xx}^2 dx \\ &+ e_R\dot{e}_R + e_v\dot{e}_v + \tilde{\mu}(t)\dot{\tilde{\mu}}(t) + \tilde{\delta}_d\dot{\tilde{\delta}}_d,\end{aligned} \tag{38}$$

where

$$\begin{aligned}\hat{T} =& \frac{2\beta\min_{x\in[0,l]}\{T\}}{\alpha} - \left(\frac{2\beta l}{\alpha} - v(t)\right)\max_{x\in[0,l]}\{T_x\} \\ &- \max_{x\in[0,l]}\{T_t\} - \frac{2\beta(c_v l + \rho A v(t))}{\alpha}.\end{aligned} \tag{39}$$

Then, employing the error dynamics (14), (38) can be rewritten as follows:

$$\begin{aligned}&\dot{V}(t) \\ &\le -\beta\{c_v l + \rho A v - \alpha v/\beta - \rho A\}\int_0^l (w_t + v w_x)^2 dx \\ &\quad - \frac{\alpha\hat{T}}{2\max_{x\in[0,l]}\{T\}}\int_0^l T w_x^2 dx - \frac{\alpha EI}{2}\int_0^l w_{xx}^2 dx \\ &\quad + \frac{h_w e_v}{(e_R + R_d)[(2\pi+1)(e_R + R_d) - R_0]}(e_v + v_d)^2 \\ &\quad - \frac{2\pi\rho n_w h_w (e_R + R_d)^2(t)}{(2\pi+1)(e_R + R_d) - R_0} e_v (e_v + v_d)^2 \\ &\quad + e_R h_w (e_v + v_d)/[(2\pi+1)(e_R + R_d) - R_0] \\ &\quad - T_0 e_v/[m_0 + 2\pi\rho n_w (e_R + R_d)(e_R + R_d + R_0)] \\ &\quad + [m_0 + 2\pi\rho n_w (e_R + R_d)^2 (e_R + R_d + R_0)] e_v \delta(t) \\ &\quad + [m_0 + 2\pi\rho n_w (e_R + R_d)^2 (e_R + R_d + R_0)] e_v \tau(t) \\ &\quad - \mu_f m_0 (e_R + R_d)^2 (e_v^2 + v_d e_v)/2 \\ &\quad + \tilde{\delta}_d\dot{\tilde{\delta}}_d(t) + \tilde{\mu}(t)\dot{\tilde{\mu}}(t).\end{aligned} \tag{40}$$

It is noted that $e_R < 0$ and $\dot{e}_R > 0$ since the radius of the rewind roller is always increased. Using (14), we obtain the following inequality.

$$e_v + v_d > 0. \tag{41}$$

And using $e_R > 0$, we obtain the following result.

$$\begin{aligned}&\frac{h_w (e_v + v_d) e_R}{[(2\pi+1)(e_R + R_d) - R_0]} \\ &\le -\frac{h_w \min\{v_d\} e_R^2}{[(2\pi+1)(e_R + R_d) - R_0] R_d}.\end{aligned} \tag{42}$$

Since the value of T_0 is sufficiently large, there exist α and β such that the following inequalities hold for all $x \in [0,l]$.

$$\frac{\beta}{\alpha} < \frac{c_v}{\rho A + c_v l + \rho A v_{\min}}, \tag{43}$$

$$2\beta l < \alpha < A v_{\min}, \tag{44}$$

$$v_{\max} < \sqrt{T_0/\rho A}, \tag{45}$$

$$\begin{aligned}&\frac{2\beta\min_{x\in[0,l]}\{T\}}{\alpha} - \left(\frac{2\beta l}{\alpha} - v_{\min}\right)\max_{x\in[0,l]}\{T_x\} \\ &\quad - \max_{x\in[0,l]}\{T_t\} - \frac{2\beta(c_v l + \rho A v_{\max})}{\alpha} > 0,\end{aligned} \tag{46}$$

where the transport velocity is assumed to be bounded, $v_{\min} < v(t) < v_{\max}$, and where the constants $v_{\min}$ and $v_{\max}$ are *a priori* known.

Using the control torque (31) and inequalities (41)-(46), (40) is rewritten as follows:

$$\dot{V}(t) \le -\lambda\{E_{beam}(t) + E_{act}(t)\}$$

$$
\begin{aligned}
&- e_R^2 \frac{h_w \min\{v_d\}}{[(2\pi+1)(e_R+R_d)-R_0]R_d} \\
&- \frac{\mu_f m_0}{2}(e_R+R_d)^2 e_v^2 - k_v e_v^2 \\
&+ \tilde{\mu}(t)\left(\dot{\hat{\mu}}_f(t) + \frac{m_0}{2}(e_R+R_d)^2 v_d e_v\right) \\
&+ \tilde{\delta}_d(t)\left(\dot{\hat{\delta}}_d(t)\right. \\
&\left. - \frac{1}{m_0 + 2\pi\rho n_w (e_R+R_d)^2(e_R+R_d+R_0)}|e_v|\right),
\end{aligned} \tag{47}
$$

where

$$
\lambda = \min\left\{ c_v - \frac{\beta}{\alpha}(\rho A + c_v l + \rho A v_{\min}), \frac{\hat{T}}{\max_{x\in[0,l]}\{T\}}, \frac{2\beta(lT_0 - \rho A l v_{\max})}{m_a(\alpha v_{\max})} \right\}. \tag{48}
$$

With the adaptation laws (34)-(35),

$$
\begin{aligned}
\dot{V}(t) \le &- \left\{ \frac{h_w \min\{v_d\}}{[(2\pi+1)(e_R+R_d)-R_0]R_d} \right\} e_R^2 \\
&- k_v e_v^2 - \frac{k_\delta}{2}\tilde{\delta}_d^2 + \frac{k_\delta}{2}\delta_d^2 - \frac{k_\mu}{2}\tilde{\mu}_f^2 + \frac{k_\mu}{2}\mu_f^2 \\
\le &- \gamma V(t) + \frac{k_\delta}{2}\delta_d^2 + \frac{k_\mu}{2}\mu_f^2
\end{aligned} \tag{49}
$$

is obtained, where

$$
\gamma = \min\left\{ \lambda, \frac{h_w \min\{v_d\}}{[(2\pi+1)(e_R+R_d)-R_0]R_d}, k_v, k_\delta, k_\mu \right\}. \tag{50}
$$

Now, a new variable is defined as follows:

$$
\omega(t) = \dot{V}(t) - \gamma V(t) - \varepsilon(t), \tag{51}
$$

where

$$
\varepsilon(t) = \frac{k_\delta}{2}\delta_d^2 + \frac{k_\mu}{2}\mu_f^2. \tag{52}
$$

From (49)-(50), it follows that $\omega(t) \le 0$. Solving (49) yields

$$
\begin{aligned}
V(t) = &V(0)e^{-\gamma t} + \int_0^t e^{-\gamma(t-\tau)}(\omega(\tau)+\varepsilon)d\tau \\
\le &V(0)e^{-\gamma t} + \frac{\varepsilon}{\lambda}(1-e^{-\gamma t}) \le V(0) + \frac{\varepsilon}{\gamma}.
\end{aligned} \tag{53}
$$

Equation (53) implies that

$$
V(t) \le V(0)e^{-\gamma t} + \frac{\varepsilon}{\gamma}(1-e^{-\gamma t}) \to \frac{\varepsilon}{\gamma} \tag{54}
$$

as $t \to +\infty$. Using Lemma 1 and Property 1, we obtain

$$
\frac{T_{\min}}{2l} w^2(x,t) \le \frac{1}{2} T_{\min} \int_0^l w_x^2(x,t)dx
$$

$$
\le \frac{V_m(t)}{\alpha - \psi} \le \frac{V(t)}{\alpha - \psi}. \tag{55}
$$

Therefore, we know that $w(x,t)$ is bounded. From (53) to (55), it is concluded that $V(t)$ is uniformly and ultimately bounded. □

Theorem 1 shows that $V(t)$ can be pushed to an arbitrarily small ball of radius ε/γ by setting small gains k_v, k_δ, and k_μ and/or a sufficiently large λ. From (54), it is also concluded that $w_x(x,t)$ and $w_t(x,t)$ are bounded. Moreover, the beam energy E_{beam} is also bounded, and using Property 1, the boundedness of $w_{xt}(x,t)$ is obtained. Therefore, from (30) and (31), the control force $f_a(t)$ is bounded. Meanwhile, the boundedness of $V(t)$ also implies the boundedness of $\hat{\mu}(t)$ and $\hat{\delta}_d(t)$, and consequently the boundedness of $\tau(t)$ too.

From Theorem 1, if the bearing friction $\mu_f(t)$ is known, and the disturbance $\delta(t)$ is ignored, the estimation process becomes unnecessary. In this case, the inequality $V(t) \le V(0)e^{-\gamma t}$ can be obtained. This implies that the exponential stability of the closed-loop system is achieved with control laws (30) and (31).

4. NUMERICAL SIMULATIONS

Numerical simulations were carried out to analyse the dyanamic responses of the rewind section. The finite difference method was employed to find an approximate solution for the PDE with the initial and boundary conditions given by (1)-(4). The convergence scheme was based on the central (for the beam span) and forward/backward (for the left/right boundary) difference methods. The system parameters are given in Table 1. Let the initial conditions of the web in the controlled span be $w(x,0) = 0.5\sin(\pi x/l)$ and $w_t(x,0) = 0$. The positive values α and β are chosen according to the inequalities (29), (41)-(44) as follows: $\alpha = 15$ and $\beta = 1.5$. The control gain $k_v = 1.5$. The adaptation gain k_μ and $k_|delta$ in (32)-(33) are selected as $k_\mu = 3$ and $k_\delta = 1.5$.

To illustrate the effectiveness of the proposed control scheme (30)-(35), two control schemes for the rewind section were considered: (i) the proposed control scheme (30)-(35) and (ii) the following control scheme

$$
f_a(t) = 0, \tag{56}
$$

$$
\tau(t) = -k_v v(t) - T(l,t)R^2 - \mu_f v_d + \dot{v}_d. \tag{57}
$$

It should be noted that the control scheme (56)-(57) was introduced without consideration of the effects of disturbances $\delta(t)$ and unknown bearing friction μ_f, where vibration control was not focused, i.e., the boundary control force $f_a(t) = 0$. The torque control $\tau(t)$ was designed only to drive the moving web to the desired velocity. Since the radius of the rewind roller R was not controlled in this case, it can be assumed that R is a constant, and conse-

Table 1. Parameters of the rewind system.

Parameter	Value
ρ	0.7 kg/m
A	0.0007 m^2
I	0.34×10^{-6} m^4
E	1.8×10^3 N/m^2
h_w	0.7×10^{-3} m
n_w	1 m
l	6 m
c_v	0.001 N·m^{2}s
m_a	1 kg
c_a	0.25 N·s/m
μ_f	2.25 N·m·s
J	2.1542 kg/m^2
R	0.2 m
T_0	100 N
$\delta(t)$	$0.5\sin(20\pi t)$ N

quently the exponential convergence of the velocity can be achieved as $e_v(t) = e_v(0)e^{-(k_v+\mu_f)}$.

As shown in Fig. 2, the transport velocity in the case of no vibration control (dashed line) converged to the desired velocity faster than in the case of vibration control (solid line), where the desired transport velocity was a typical velocity profile (dotted line) widely used in the industry. The settling time in the case of no vibration control was 1.5 seconds (with $k_v = 40$), whereas it was 2 seconds in the case of vibration control. This can be explained as follows. A large control gain k_v in the control torque law (54) can be set in the case of no vibration control; meanwhile, in the case of vibration control, as shown in (50), the coefficient of convergence γ cannot become equal to a large value of control gain k_v in control torque law (31).

In the case of no vibration control, the transverse vibration was suppressed by a viscous damping force. However, the time for suppression was too long, i.e., it took more than 20 seconds, as shown in Fig. 3. This result indicates that vibrations of the moving web must be controlled by an active control scheme.

The result in the case of vibration control using the control scheme (30)-(35) is shown in Fig. 4. It is obvious that good performance has been obtained. The times required for vibration suppression in the acceleration, constant speed, and deceleration periods of the velocity profile were less than two seconds. Vibration occurring during the transition between the acceleration and constant speed periods was suppressed within one second. This also happened at the transition between the constant speed and deceleration periods. It was shown that the amplitude of the vibration in the case of no vibration control was considerably larger than that in the case of vibration control.

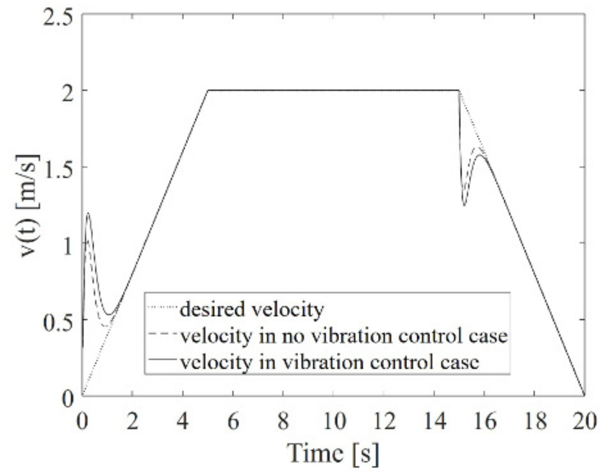

Fig. 2. Comparison of transport velocity tracking in the cases of no vibration control (dashed line) and vibration control (solid line).

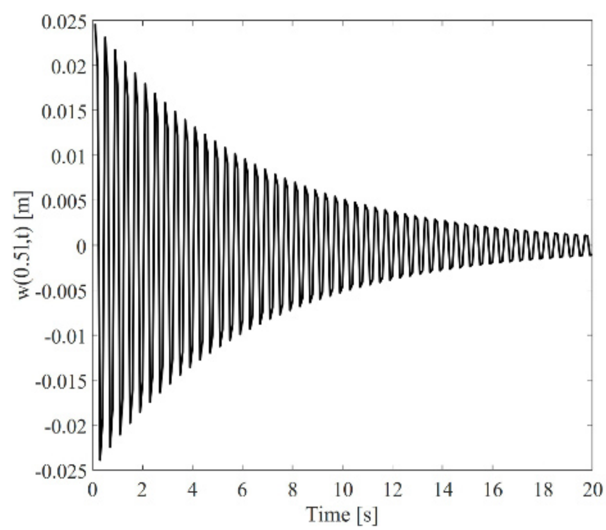

Fig. 3. Transverse displacement of the web at $x = l/2$ in the case of no vibration control.

Fig. 5 shows that the radius approaches the desired radius. In Figs. 6 and 7, the convergences of the unknown bearing friction coefficient and the bound of the disturbance to constant values are demonstrated, but they are not true values. However, this issue is not the focus of this paper.

5. CONCLUSIONS

In this paper, an active control scheme was developed for the rewind section in an R2R system. The control scheme was able to suppress the transverse vibration of the moving web, to maintain the transport velocity, and to

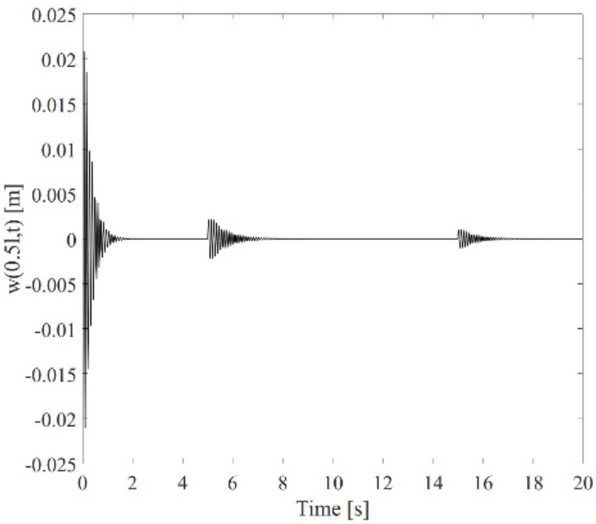

Fig. 4. Transverse displacement of the web at $x = l/2$ in the case of vibration control.

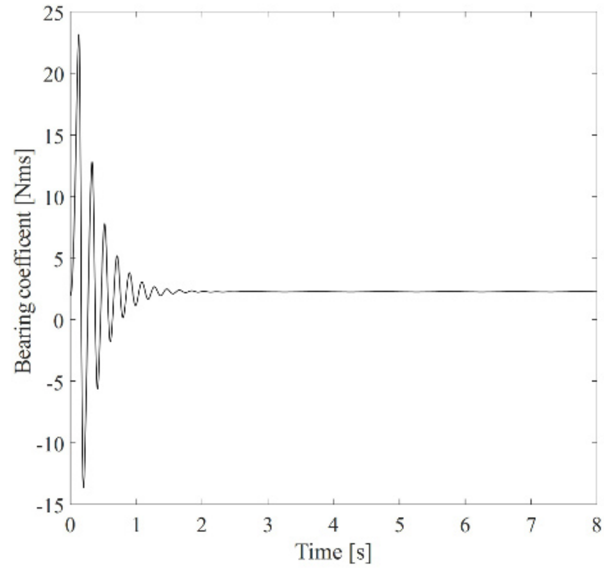

Fig. 6. Convergence of the bearing coefficient $\hat{\mu}_f$.

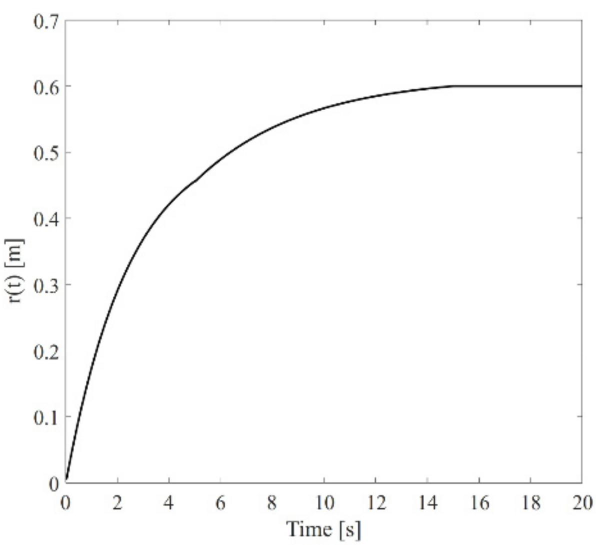

Fig. 5. Radius of the rewind roller.

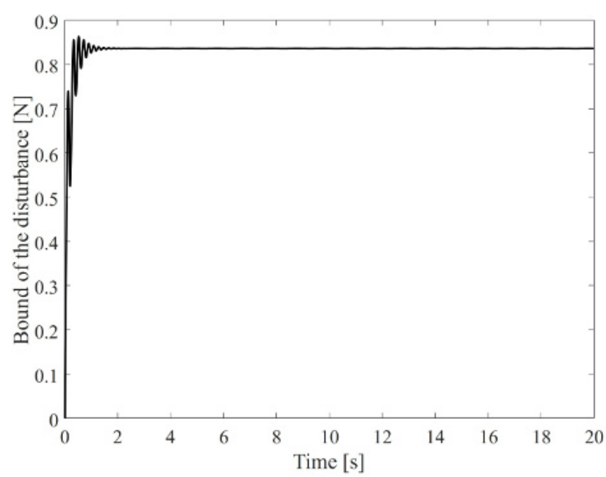

Fig. 7. Convergence of the bound of the disturbance $\hat{\delta}_d(t)$.

drive the radius of the rewind roller to its desired value. The dynamic effects of the time-varying tension were incorporated into the control design. Adaptation laws were derived to cope with the unknown bearing friction coefficient and the unknown disturbance resulting from the variations of the rotating elements on the drive motor side. The Lyapunov energy-based method was used to prove the robust stability of the rewind section in the sense that all of the signals were uniformly and ultimately bounded. In the next stage of this research, the proposed control algorithm will be tested in an industry control system in order to verify the effectiveness of the control method in the industry.

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